

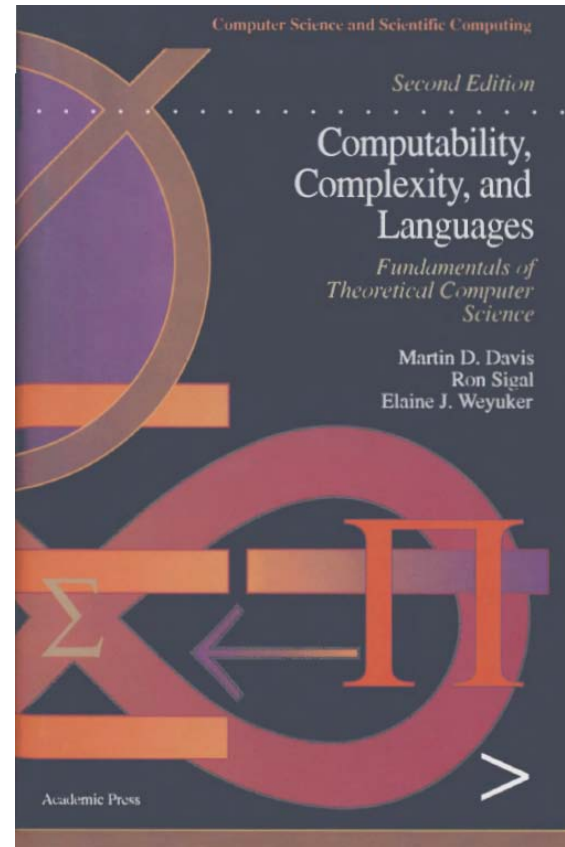
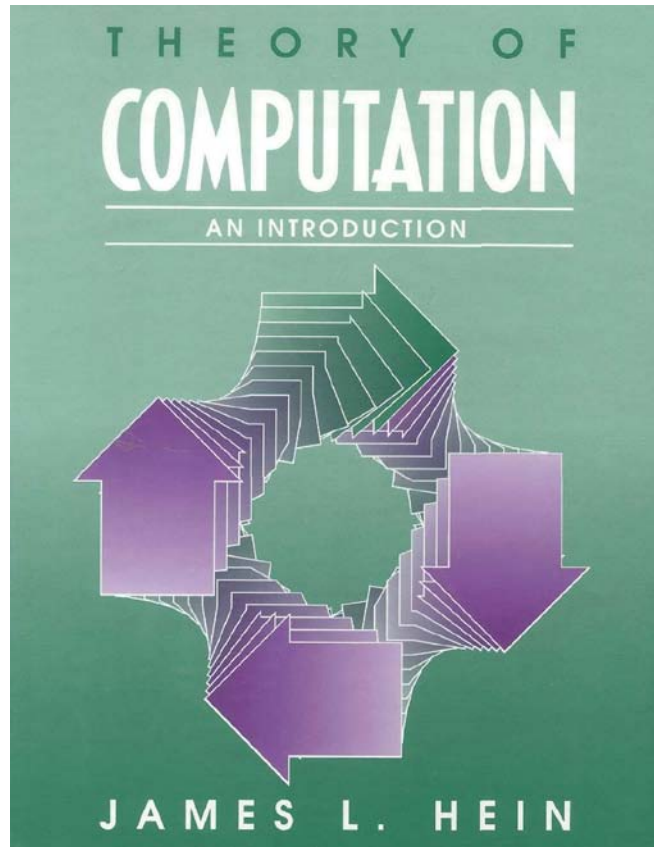
# Theory of Computation

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Lecture 02

# Books

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# PowerPoint

<http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767>

The screenshot displays a web interface for Benha University. At the top, there is a blue header with the university logo, the name 'Benha University', and a welcome message for 'Ahmed Hassan Ahmed Abu El Atta' with a 'Log out' link. Below the header, a navigation menu on the left lists various options like 'Home', 'My C.V.', 'About', 'Courses', etc. The main content area shows the user's current location: 'Home/Courses/Automata and Formal Languages', with a 'Back To Courses' link. The course title is 'Ass. Lect. Ahmed Hassan Ahmed Abu El Atta :: Course Details: Automata And Formal Languages'. There are links for 'add course' and 'edit course'. A table lists course details: Course name (Automata and Formal Languages), Level (Undergraduate), Last year taught (2018), and Course description (Not Uploaded). Below this is a 'Course password' field. At the bottom, there are sections for 'Course files', 'Course URLs', 'Course assignments', and 'Course Exams & Model Answers', each with an 'add' link. On the right side, there are social media icons for Google, Facebook, LinkedIn, and others, along with an '(edit)' link.

Course name	Automata and Formal Languages
Level	Undergraduate
Last year taught	2018
Course description	Not Uploaded
Course password	
Course files	<a href="#">add files</a>
Course URLs	<a href="#">add URLs</a>
Course assignments	<a href="#">add assignments</a>
Course Exams & Model Answers	<a href="#">add exams</a>

# Turing Machine

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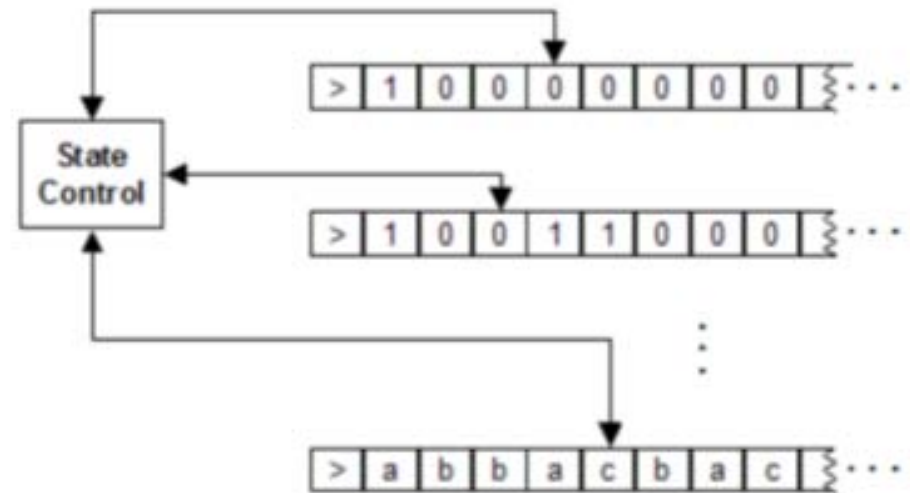
# Agenda

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- Multi-Tape Turing machine
- EXAMPLE 1 (*Multiplying Natural Numbers*)
- Move one cell to right
- Turing machines are *equivalent* in power
- EXAMPLE 2

# Multi-Tape Turing machine

- A multi-tape Turing machine has two or more tapes with corresponding tape heads.
- It's important to note that all these Turing machines are *equivalent* in power.
- In other words, any problem solved by one type of Turing machine can also be solved on any other type of Turing machine.



# EXAMPLE 1 (*Multiplying Natural Numbers*)

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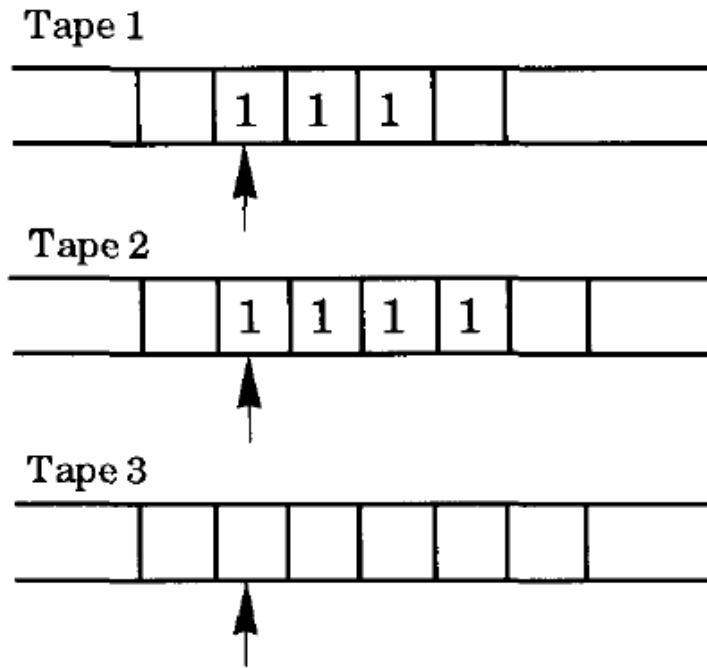
- Suppose we want to construct a Turing machine to multiply two natural numbers, each represented as a unary string of ones.
- We'll use a three-tape Turing machine, where the first two tapes hold the input numbers and the third tape will hold the answer.

$\langle i, \langle a, b, c \rangle, \langle x, y, z \rangle, \langle R, L, S \rangle, j \rangle$ .

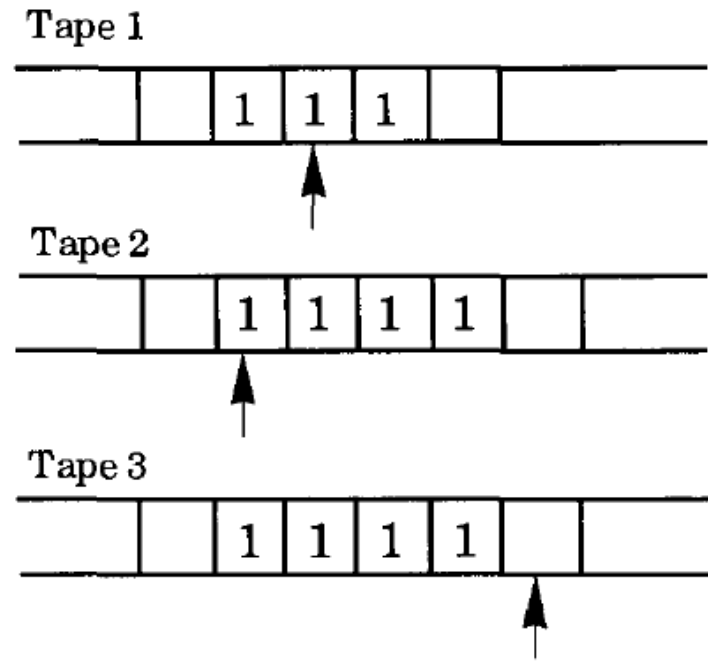
For example, the diagrams show the contents of the three tapes before the computation of 3.4 and before the start of the second of the three additions

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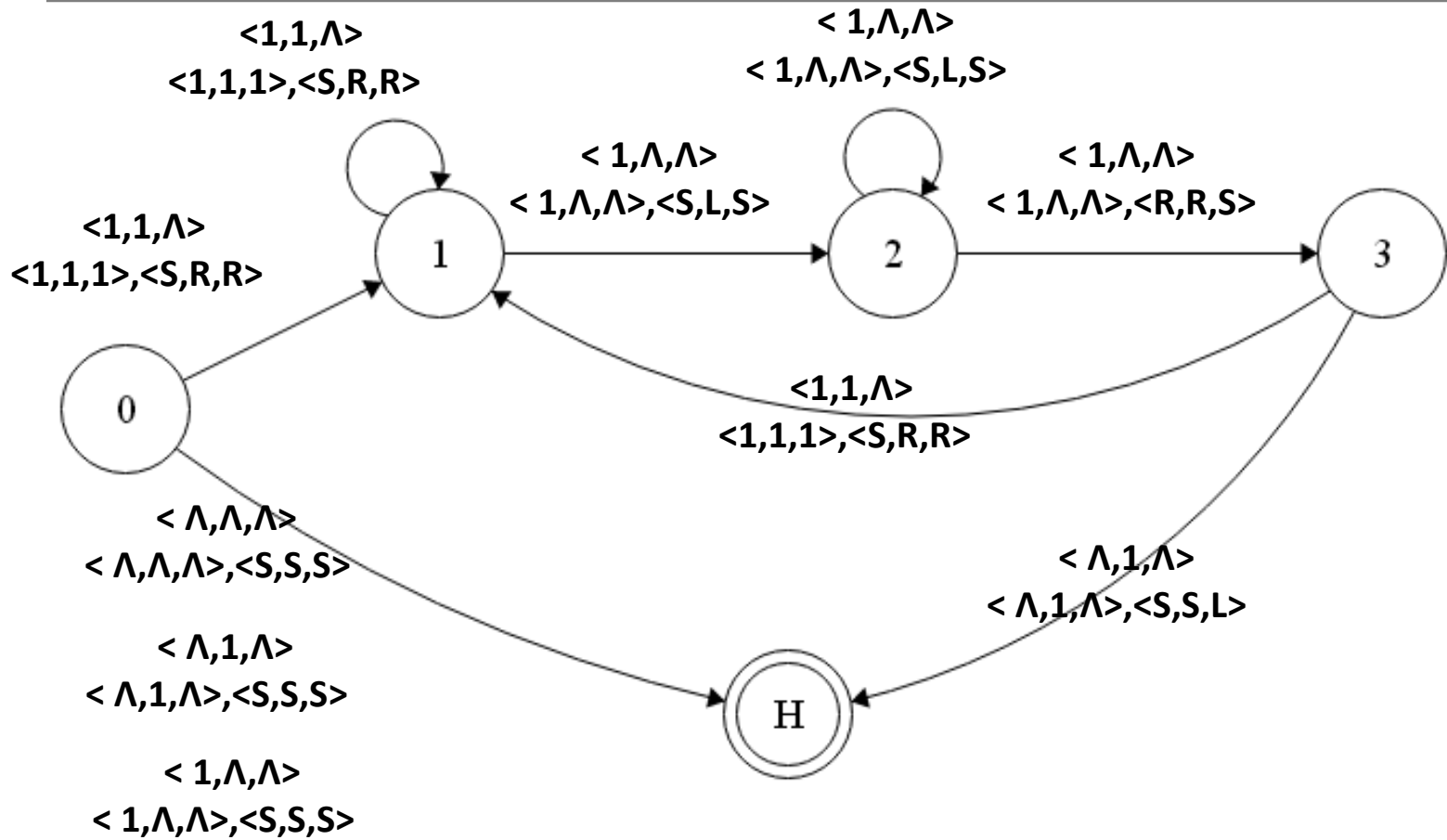
TAPES BEFORE STARTING TO COMPUTE 3.4



TAPES BEFORE STARTING THE SECOND ADDITION

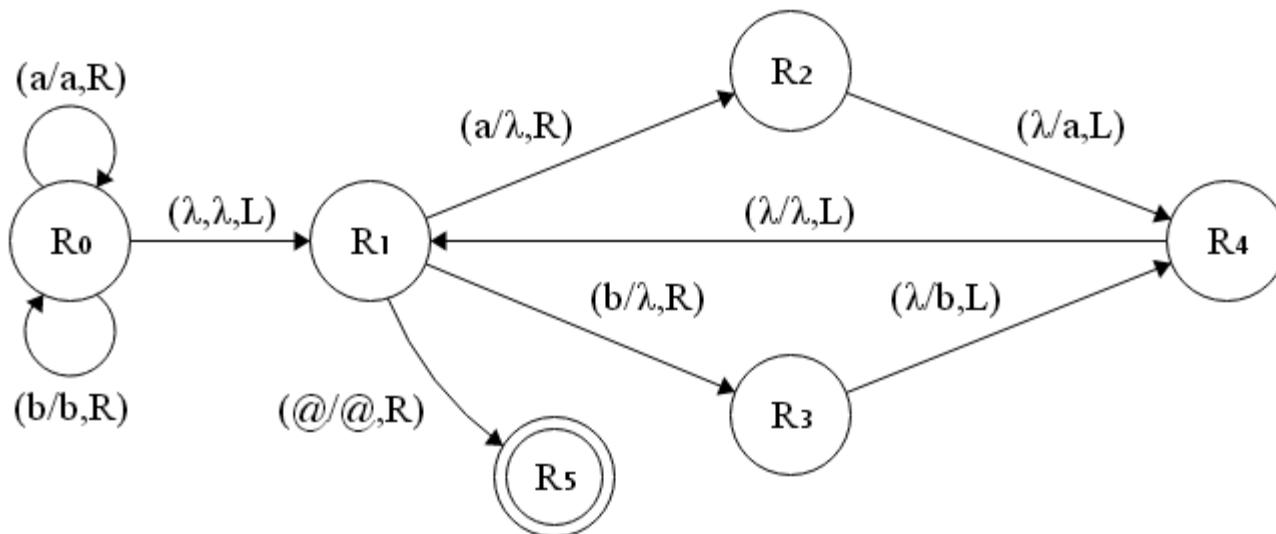
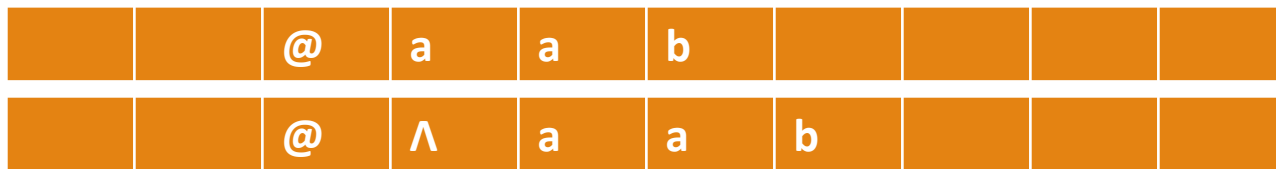






# Move one cell to right

Construct a Turing machine to move an input string over  $\{a, b\}$  to the right one cell position. Assume that the tape head is at the left end of the input string if the string is nonempty. The rest of the tape cells are blank. The machine moves the entire string to the right one cell position, leaving all remaining tape cells blank.



# Turing machines are *equivalent* in power

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- Assume that we have a Turing machine **T** that has two tapes, each with a single tape head.
- We'll describe a new single-tape, single-head machine **M** that will start with its tape containing the two nonblank portions taken from the tapes of **T**, separated by a new tape symbol

@

# Turing machines are *equivalent* in power

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Whenever **T** executes an instruction (which is actually a pair of instructions, one for each tape), **M** simulates the action by performing two corresponding instructions, one instruction for the left side of **@** and the other instruction for the right side of **@**.

# Turing machines are *equivalent* in power

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Since **M** has only one tape head, it must chase back and forth across **@** to execute instructions.

So it needs to keep track of the positions of the two tape heads that it is simulating.

One way to do this is to place a position marker **.** in every other tape cell.

$$\dots \Lambda \cdot \# \cdot a \cdot b \wedge c \cdot @ \cdot x \cdot y \wedge z \cdot w \cdot \# \cdot \Lambda \dots$$

# Turing machines are *equivalent* in power

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To indicate a current cell, we'll write the symbol

**^**

in place of **.** in the adjacent cell to the right of the current cell for the left tape and to the adjacent cell to the left of the current cell for the right tape.

For example, if the two tapes of **T** contain the strings **abc** and **xyzw**, with tape heads pointing at **b** and **z**, then the tape for **M** has the following form, where the symbol **#** marks the left end and the right end of the relevant portions of the tape:

$$\dots \Lambda \cdot \# \cdot a \cdot b \wedge c \cdot @ \cdot x \cdot y \wedge z \cdot w \cdot \# \cdot \Lambda \dots$$

# Turing machines are *equivalent* in power

$\langle i, \langle a, x \rangle, \langle 0, 1 \rangle, \langle R, R \rangle, j \rangle$

		^	x	y	z				
--	--	---	---	---	---	--	--	--	--

		^	a	b	c				
--	--	---	---	---	---	--	--	--	--

.	#	.	a	^	b	.	c	.	@	^	x	.	y	.	z	.	#	.	
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--

$\langle i, a, 0, R, k \rangle$

$\langle k, ^, ., R, k \rangle$

$\langle k, ., ^, R, h \rangle$

$\langle k, \text{anyElse}, \text{anyElse}, R, k \rangle$

$\langle h, ^, ., R, q \rangle$

$\langle h, \text{AnyElse}, \text{AnyElse}, R, h \rangle$

$\langle q, x, 1, R, p \rangle$

$\langle p, ., ^, L, g \rangle$

$\langle g, ^, ^, L, j \rangle$

$\langle g, \text{AnyElse}, \text{AnyElse}, L, g \rangle$

# Example 2

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Construct Turing machine to add 3 to a natural number represented as a binary string.



# Example 2

Construct Turing machine to add 3 to a natural number represented as a binary string.

